Ist. Mat. I - C/A
2/3/23

(39) $\int e^{-x^2}\cdot x^3 = -\frac{1}{2}\cdot\int\left(-2xe^{-x^2}\right)\cdot x^2$

$$= -\frac{1}{2}e^{-x^2}\cdot x^2 + \frac{1}{2}\int e^{-x^2}\cdot 2x$$

$$= -\frac{1}{2}e^{-x^2}\cdot x^2 - \frac{1}{2}e^{-x^2}$$

$$\boxed{\begin{array}{l}\cos^2+\sin^2=1\\ \cosh^2-\sinh^2=1\end{array}}$$

(48) $\int \frac{x^3}{\sqrt{2-x^2}}$

$x=\sqrt{2}\cdot\sin(t)$ $\quad dx=\sqrt{2}\cdot\cos(t)$ $\quad t=\arcsin\left(\frac{x}{\sqrt{2}}\right)$

$$\int\frac{2\sqrt{2}\sin^3(t)}{\sqrt{2}\cos(t)}\cdot\sqrt{2}\cos(t) = 2\sqrt{2}\int\sin^3(t)$$

$$= 2\sqrt{2}\int\left(\sin(t)-\cos^2(t)\sin(t)\right)$$

$$= 2\sqrt{2}\cdot\left(-\cos(t)+\frac{1}{3}\cos^3(t)\right) \qquad \left(\sqrt{2}\cos(t)=\sqrt{2-x^2}\right)$$

$$= -2\sqrt{2-x^2}+\frac{1}{3}\left(\sqrt{2-x^2}\right)^3$$

$$= -\frac{1}{3}\sqrt{2-x^2}\left(6-(2-x^2)\right) = -\frac{1}{3}(4+x^2)\sqrt{2-x^2}$$

Oppure per parti:

$$\int \frac{x^3}{\sqrt{2-x^2}} = \int \frac{1}{2}\cdot(-2x)\cdot\frac{1}{\sqrt{2-x^2}}\,(-x^2)$$

$$= \sqrt{2-x^2}\cdot(-x^2) - \int \sqrt{2-x^2}\cdot(-2x)$$

$$= \sqrt{2-x^2}\cdot(-x^2) - \frac{2}{3}\cdot\left(2-x^2\right)^{\frac{3}{2}}$$

$$= -\frac{1}{3}\sqrt{2-x^2}\left(3x^2+2(2-x^2)\right) = -\frac{1}{3}(4+x^2)\sqrt{2-x^2}$$

(49) $\displaystyle\int \frac{x^2}{\sqrt{1+x^2}}$ $\qquad x = \sinh(t) \qquad dx = \cosh(t)\,dt$

$$\int \frac{\sinh^2(t)}{\cosh(t)}\cdot\cosh(t)\,dt = \int \sinh^2(t)\,dt$$

$$= \int \frac{e^{2t}-2+e^{-2t}}{4} = \frac{e^{2t}}{8} - \frac{1}{2}t - \frac{e^{-2t}}{8}$$

$$= \frac{1}{4}\sinh(2t) - \frac{1}{2}t\,\Big|_{t=\operatorname{arcsinh}(x)}$$

(51) $\displaystyle\int \frac{\sqrt{x^2-2}}{x^2}$ $\qquad x = \sqrt{2}\cdot\cosh(t), \quad dx = \sqrt{2}\cdot\sinh(t)\,dt$

$$= \int \frac{\sqrt{2}\cdot\sinh(t)}{2\cosh^2(t)}\cdot\sqrt{2}\sinh(t)\,dt = \int \frac{\sinh^2(t)}{\cosh^2(t)}$$

$$= \int \frac{e^{2t}-2+e^{-2t}}{e^{2t}+2+e^{-2t}} \qquad e^{2t}=y \qquad t=\frac{1}{2}\log(y)$$

$$dt = \frac{1}{2y}\,dy$$

$$= \int \frac{y-2+\frac{1}{y}}{y+2+\frac{1}{y}} \cdot \frac{1}{2y} = \frac{1}{2}\int \frac{(y-1)^2}{y(y+1)^2}\,dy = \dots$$

(56) $\displaystyle\int_0^1 (1-x^2)^{3/2}\,dx$

$x=\sin(t)$ $\qquad dx=\cos(t)\,dt$

$x=0 \longrightarrow t=0$

$x=1 \longrightarrow t=\pi/2$

$$= \int_0^{\pi/2} \cos^3(t)\cdot\cos(t)\,dt = \int_0^{\pi/2} \cos^4(t) = \dots$$ per duplicazione due volte

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<u>Integrali impropri</u>

(72) $\displaystyle\int_0^1 x\cdot\log(x)\,dx$

continua su $(0,1]$ ma si estende con continuità in $x=0$

$\Longrightarrow$ Converge

$$\int_0^1 x\cdot\log(x) = \frac{1}{2}x^2\cdot\log(x)\Big|_0^1 - \int_0^1 \frac{1}{2}x^2\cdot\frac{1}{x}\,dx$$

$$= \frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 \Big|_0^1 = 0 - \frac{1}{4} - (0-0) = -\frac{1}{4}$$

(73) $\displaystyle\int_0^{+\infty} \underbrace{\frac{1}{\sqrt{x}}\cdot e^{-\sqrt{x}}}_{\text{continua su } (0,+\infty)}$

$$= \int_0^1 \frac{1}{\sqrt{x}}\cdot \underbrace{e^{-\sqrt{x}}}_{\text{limitata tra } 1/2 \text{ e } 1} + \int_1^{+\infty} \frac{1}{\sqrt{x}}\cdot e^{-\sqrt{x}}$$

Non int. ma ≤ 1

$\leq \frac{1}{x^\alpha}$ $\forall \alpha > 1$

integrabile

OK

OK

$\displaystyle\int_0^1 \frac{1}{x^\alpha}$ $\alpha < 1$

OK

$$\int_0^{+\infty} \frac{1}{\sqrt{x}}\cdot e^{-\sqrt{x}} = -2\cdot e^{-\sqrt{x}} \Big|_0^{\infty} = 0-(-2) = 2$$

——— o ———

$$\int_1^{+\infty} f(x)\cdot g(x)\,dx$$

Se so che

- $f(x), g(x) \geq 0$
- $f(x) \leq C$
- $\displaystyle\int_1^{\infty} g(x)\,dx < +\infty$

$\Longrightarrow \displaystyle\int_1^{\infty} f(x)\cdot g(x)\,dx < +\infty$

(74) $$\int_0^1 \underbrace{\frac{\sin(x)}{x^2}}\,dx$$

continua su $(0,1]$

vicino a 0 va come $\frac{1}{x}$

(compresa tra $k\cdot\frac{1}{x}$ e $k\cdot\frac{1}{x}$)

Ma $\int_0^1 \frac{1}{x} = +\infty \implies \int_0^1 \frac{\sin(x)}{x} = +\infty$

(75) $$\int_0^\infty \underbrace{e^{-2x}\cdot \sin(\underset{\leq\; e^{-x}}{e^{-x}})}\,dx$$

$$e^{-3x}$$

$$\wedge$$

$$\frac{1}{x^\alpha} \quad \forall \alpha > 1 \implies \text{convergente}$$

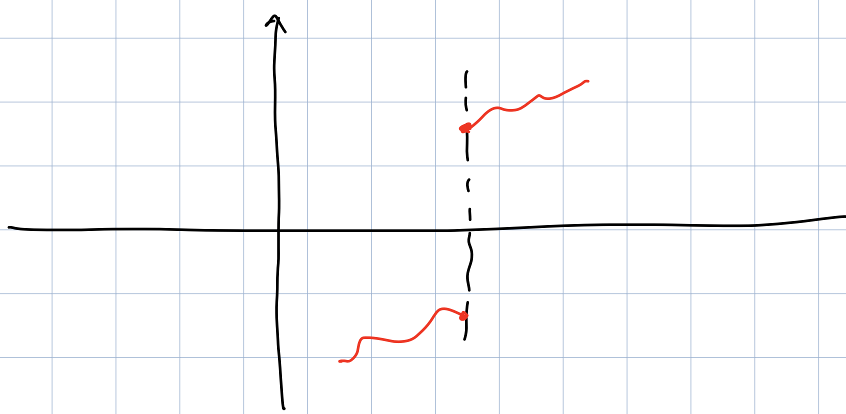
(76) $$\int_2^4 \underbrace{\arctan\left(\frac{x}{x-3}\right)}\,dx$$

$\Rightarrow$ definita su $[2,3) \cup (3,4]$

$$= \int_2^3 \arctan\left(\frac{x}{x-3}\right) + \int_3^4 \arctan\left(\frac{x}{x-3}\right)$$

si estende per continuità su $[2,3]$ ponendo valore $-\pi/2$ in $x=3$

. . . . $[3,4]$ valore $\pi/2$ in $x=3$



$$\int_2^4 \arctan\left(\frac{x}{x-3}\right) \qquad t = x-3 \qquad \begin{matrix} x=4 & t=1 \\ x=2 & t=-1 \end{matrix}$$

$$\int_{-1}^{1} \arctan\left(\frac{t+3}{t}\right) = \int_{-1}^{1} \arctan\left(1+\frac{3}{t}\right) = \dots$$

(43)
$$\int_0^{\pi/2} \frac{\sin(x) + 2\cos(x)}{2+\sin(x)}\, dx$$

$$t = \tan\left(\frac{x}{2}\right) \qquad \sin(x) = \frac{2t}{1+t^2} \qquad \cos(x) = \frac{1-t^2}{1+t^2}$$

$$x = 2\arctan(t) \qquad x = 0 \longrightarrow t = 0$$

$$dx = \frac{2\,dt}{1+t^2} \qquad x = \pi/2 \longrightarrow t = \tan(\pi/4) = 1$$

$$\int_0^1 \frac{\frac{2t}{1+t^2} + 2\cdot\frac{1-t^2}{1+t^2}}{2+\frac{2t}{1+t^2}} \cdot \frac{2dt}{1+t^2} = 2\int_0^1 \underbrace{\frac{1+t-t^2}{(t^2+t+1)(t^2+1)}}_{\frac{at+b}{t^2+t+1} + \frac{ct+d}{t^2+1}}\, dt$$

$$= \dots = 4\int_0^1 \frac{t}{t^2+t+1} + 2\int_0^1 \frac{1-2t}{t^2+t+1}$$

fare conto esplicito

$$4\int_0^1 \frac{t}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}} = 4\int_0^1 \frac{t+\frac{1}{2}}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}} - 2\int \frac{1}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= 4\cdot\frac{1}{2}\cdot\log\left(t^2+t+1\right)\Big|_0^1 - \frac{8}{3}\int_0^1 \frac{1}{\left(\frac{2t+1}{\sqrt{3}}\right)^2+1}$$

$$= 2\cdot\log\left(t^2+t+1\right)\Big|_0^1 - \frac{8}{3}\cdot\frac{\sqrt{3}}{2}\arctan\left(\frac{2t+1}{\sqrt{3}}\right)\Big|_0^1$$

$$= 2\cdot\log(3) - 0 - \frac{4}{\sqrt{3}}\cdot\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = 2\log(3) - \frac{2\pi}{3\sqrt{3}}$$